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Influence of Aerodynamic Loads on Flight Trajectory of Spinning Spherical Projectile

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The effect of lift and drag forces and aerodynamic moment on the flight of a spinning projectile in the atmosphere is investigated. The influence of the spin rate of the projectile on the range and height of flight is also studied, taking into consideration the progressive decay in the spin rate during the flight. The governing equations of motion of the projectile have been solved numerically, and the results are presented graphically. The effect of various parameters such as angle of projection, initial velocity, and spin on the flight of projectiles are plotted. From the results it can be concluded that the lift and drag forces and the aerodynamic moment, which are dependent on the spin rate of the projectile, influence the range and height of the trajectory significantly. Only planar motion of the projectile is considered.

Introduction

PROJECTILE motion in atmosphere has a wide variety of applications in the military, sports, etc. Investigations of the motion of bullets and shells have been done in great detail by McShane et al.,[1] Farrar and Leeming,[2] and references therein. For nonmilitary applications, the behavior of projectiles in flight has received comparatively limited attention. Some references in this connection are Hart and Croft,[3] de Mestre,[4] Stewart-Townend,[5] de Mestre and Catchpole,[6] de Mestre,[7] and Jorgensen,[8] to name a few.

Bentley et al.[9] experimentally investigated the swing of a cricket ball. Golf ball aerodynamics has been studied by Davies[10] and Bearman and Harvey.[11] Mehta[12] also studied aerodynamics of sports balls. Projectile motion in a resisting medium has been investigated by Murphy[13] and Parker.[14] Recently Suzuki and Inooka[15] studied a golf-swing robot model utilizing shaft elasticity.

From a survey of the literature in the area, it is clear that studies are scarce on the maximum range and the heights of the projectile flight in the presence of the changing drag and lift forces, as well as on the aerodynamic moment resisting the spin acting on the projectile during the flight in atmosphere. Bearman and Harvey[11] investigated the problem, ignoring the aerodynamic moment, by experimentally obtaining the lift and drag parameters in the presence of constant spin rates.

When a body is projected in atmosphere, it is subjected to forces due to drag, which directly opposes the motion, and to lift, which acts at right angles to the drag and generally in a normal direction to velocity. These two forces are invariably present in all of the projectiles in atmosphere such as those in sports (golf, baseball, tennis, football), missiles, etc. Note that without the force of the air on the object, the object would travel along a very different path than it actually does. When there is no air resistance, it is well known that the projectile path is parabolic. However, the presence of spin, which changes the lift and drag forces and also introduces aerodynamic moments, produces spectacular modifications in the projectile flight path.

The present paper analyzes the trajectory to study the influence of nonlinear lift and drag forces and the aerodynamic moments on the projectile flight. The decay of the spin as it moves changes its drag and lift considerably and in turn gives different range and height. Note that this path depends on the initial velocity, spin, and angle of projection of the projectile.

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Mathematical Background

For a projectile launched in atmosphere, three forces are important, namely, gravity, drag, and lift. Let a body be driven from the ground with linear initial velocity v_0 and the initial angle of projection ϕ_0.

In rectangular absolute coordinates,

$$\frac{d^2\bar{r}}{dt^2} = \frac{d\bar{V}}{dt} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} \tag{1}$$

The drag force D and lift force L are given by de Mestre[7] as

$$\bar{D} = -\tfrac{1}{2}\rho A v_0^2 C_D \hat{\tau} \tag{2}$$

$$\bar{L} = \tfrac{1}{2}\rho A v_0^2 C_L \hat{\eta} \tag{3}$$

where ρ and A are density of air and cross-sectional area of the projectile and C_D and C_L are drag and lift coefficients. Moreover, $\hat{\tau}$ and $\hat{\eta}$ are unit moving vectors along tangential and normal directions of the velocity $\hat{v}$ and are given by

$$\hat{\tau} = \cos\phi\hat{i} + \sin\phi\hat{j} \tag{4}$$

$$\hat{\eta} = -\sin\phi\hat{i} + \cos\phi\hat{j} \tag{5}$$

The governing equations of motion with the nonlinear lift and drag forces for a projectile are

$$m\frac{d\bar{V}}{dt} = -m\bar{g} + \bar{D} + \bar{L} \quad \text{or}$$

$$m\frac{d^2\bar{r}}{dt^2} = -mg\hat{j} - \frac{1}{2}\rho A v^2 C_D \hat{\tau} + \frac{1}{2}\rho A v^2 C_L \hat{\eta} \tag{6}$$

where m and g are mass of the projectile and acceleration due to gravity, respectively. Now substituting Eqs. (1), (4), and (5) in (6) and equating coefficients of $\hat{i}$ and $\hat{j}$, we have accelerations along the x and y directions as

$$a_x = \frac{d^2x}{dt^2} = -\frac{\rho A}{2m}v^2(C_D\cos\phi + C_L\sin\phi) \tag{7}$$

$$a_y = \frac{d^2y}{dt^2} = -g - \frac{\rho A}{2m}v^2(C_D\sin\phi - C_L\cos\phi) \tag{8}$$

To compute the trajectory, the initial spin, launch angle, and launch velocity need to be known. During the path the spin parameter and the Reynolds number will change the lift and drag coefficients, which is discussed next.

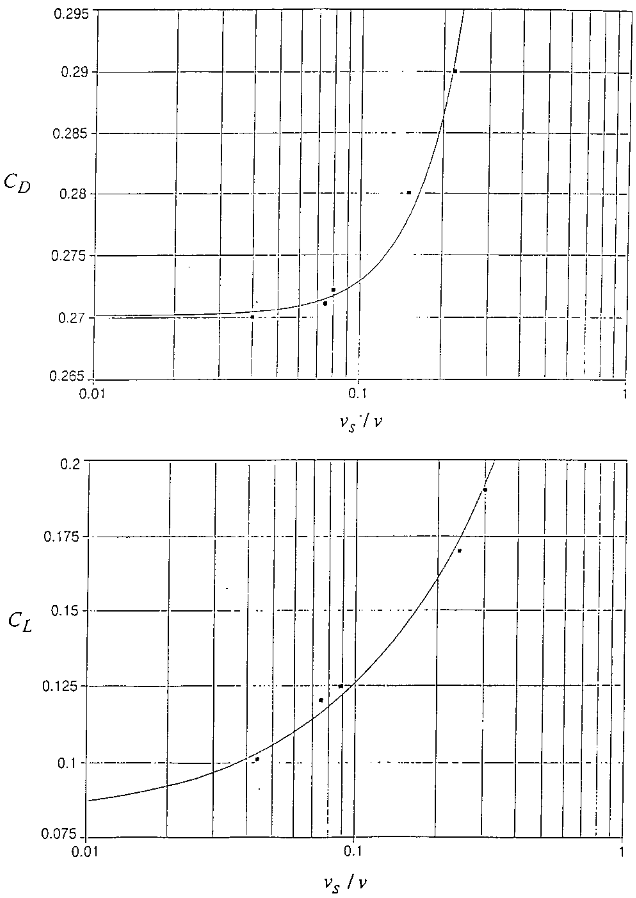

Fig. 1 Drag and lift coefficients as functions of the spin parameter.

Drag and Lift Coefficients from the Spin Parameter

If N is the spin rate of the ball, measured in revolutions per minute, its peripheral speed or equatorial speed v_s is given by

$$v_s = \pi N d/60 \tag{9}$$

where d is the diameter of the ball. Then the velocity ratio v_s/v will be termed the spin parameter. The typical value of Reynolds number for a golf ball as given by de Mestre[7] is 1.9×10^5. Bearman and Harvey[11] experimentally obtained the drag and lift coefficients, C_D and C_L, respectively. Curves are fitted to C_D and C_L as functions of the spin parameter and are given by

$$C_D = \exp\left[a + b(v_s/v)^{2.5}\right]$$

$$a = -1.3088142, \qquad b = 3.2356912 \tag{10}$$

$$C_L = \sqrt{a + b\exp(v_s/v)}$$

$$a = -0.079355168, \qquad b = 0.086091392 \tag{11}$$

which are shown in Fig. 1. Recently, Smits and Smith[16] presented extensive experimental results on the lift and drag coefficients in the presence of spin decay. They also proposed some mathematical models for the spin rate decay. However, in view of the large variations in results with respect to the various parameters, the mathematical models proposed are quite empirical. For the range of Reynolds numbers considered, the Bearman and Harvey[11] results agree with the results of Smits and Smith,[16] and the fitted curves in Eqs. (10) and (11) are quite reasonable.

Spin Decay Model

The angular deceleration α is given by the ratio of friction moment to moment of inertia I, of the ball about an axis passing through the ball center; that is,

$$\alpha = 0.5\rho A(\pi N d/60)^2(d/2)(C_D) \Big/ \tfrac{2}{5} m r^2$$

where the numerator corresponds to the moment causing the spin decay. It is the product of the skin-friction force due to the rotational velocity of the ball, which is continuously distributed around the ball periphery, and the radius. This expression can be simplified to

$$\alpha = (r\rho A\pi^2/720m)N^2(C_D) \tag{12}$$

Even though the friction moment on the ball must arise due to the skin friction, only the skin friction due to the ball spin is responsible for the spin decay and not the skin friction drag due to the translational velocity of the ball. Moreover, it is quite cumbersome to separate the skin-friction drag and the form drag, both of which change continuously with the velocity and the spin itself. The pressure distribution around the sphere will influence the shear stresses depending on the condition of the boundary layer. This in turn influences pressure drag. They are all interrelated. Moreover, according to Smits and Smith[16] the C_D varies linearly with the spin rate, and spin rate decay (SRD) is proportional to the spin rate. Hence, taking the angular deceleration (or SRD) proportional to C_D seems quite reasonable. Moreover, this approach has been used by other researchers, such as Adair,[17] working in similar areas.

By the denoting of the initial spin as N_{old} at each time interval, the new spin N_{new} is obtained by the following equation:

$$N_{\text{new}} = N_{\text{old}} - \alpha t = N_{\text{old}} - (r\rho A\pi^2/360m)C_D(N_{\text{old}})^2 t \tag{13}$$

Clearly Eq. (13) is iterative in nature. This iteration is continued until the spin converges at each time interval, which is used to evaluate the C_D and C_L at each time.

Results and Discussion

In the present work the solution for the maximum range and flight paths have been found numerically from Eqs. (7) and (8) by computing x and y components of the trajectory by a fourth-order Runge–Kutta method taking a time step of 0.001 in SIMULINK model, as shown in Fig. 2. In the SIMULINK model, Eqs. (10) and (11) are evaluated at each time interval to obtain the corresponding values of C_D and C_L depending on the spin parameter. The iteration in Eq. (13) is continued till the spin converges at each time interval, which is used to evaluate the C_D and C_L at each time.

The well-known trajectory paths for angle of projection equal to 5, 15, 30, 45, 60, and 75 deg in vacuum, without air resistance and spin, are shown in Fig. 3. For all of the cases, the initial velocity of projection has been taken as 70 m/s.

Figure 4 shows the various flight paths for the same parameters as in Fig. 3 but with air resistance. The initial spin parameter has been taken as 2500 rpm (Ref. 11). It is seen that the value of the spin decays to about half of its value when it reaches the ground. Note that the maximum range changes a bit when the spin decay is incorporated for larger angles of projection. However, for smaller angles of projection, for example, for 5 deg, the maximum range is 110.8 m when considering the spin decay, whereas without decay, it is 118.36 m.

Considering the air resistance and the other aerodynamic parameters, we attain the maximum range for the initial angle of projection of around 35–40 deg. This has been confirmed by numerical experiments for various sets of parameters. From Figs. 3 and 4, it is clear that the flight paths where aerodynamic forces and moments are included are not symmetric parabolas as in a vacuum. Moreover in a vacuum, the ranges are identical for the initial angles of projection of 45 deg $\pm\ \phi$. However, this is not true in case of aerodynamic resistance, as can be seen in Fig. 4.

Figures 5 and 6 show the maximum range and maximum height of the flight, respectively, as the angle of projection changes for different typical values of initial velocity of projection from 30 to 70 m/s at an interval of 10 m/s. As reported earlier, we see again from Fig. 5 that the maximum range is obtained for the angle of projection of around 35–40 deg.

In Figs. 7 and 8, results for different values of spin rates are presented. Figure 7 shows the plot of maximum range against angle of projection for the constant initial velocity of 57.9 m/s for different spin rates. The results of Bearman and Harvey[11] for spin rate of 3500 rpm are also given in Fig. 7. Note that their results give larger

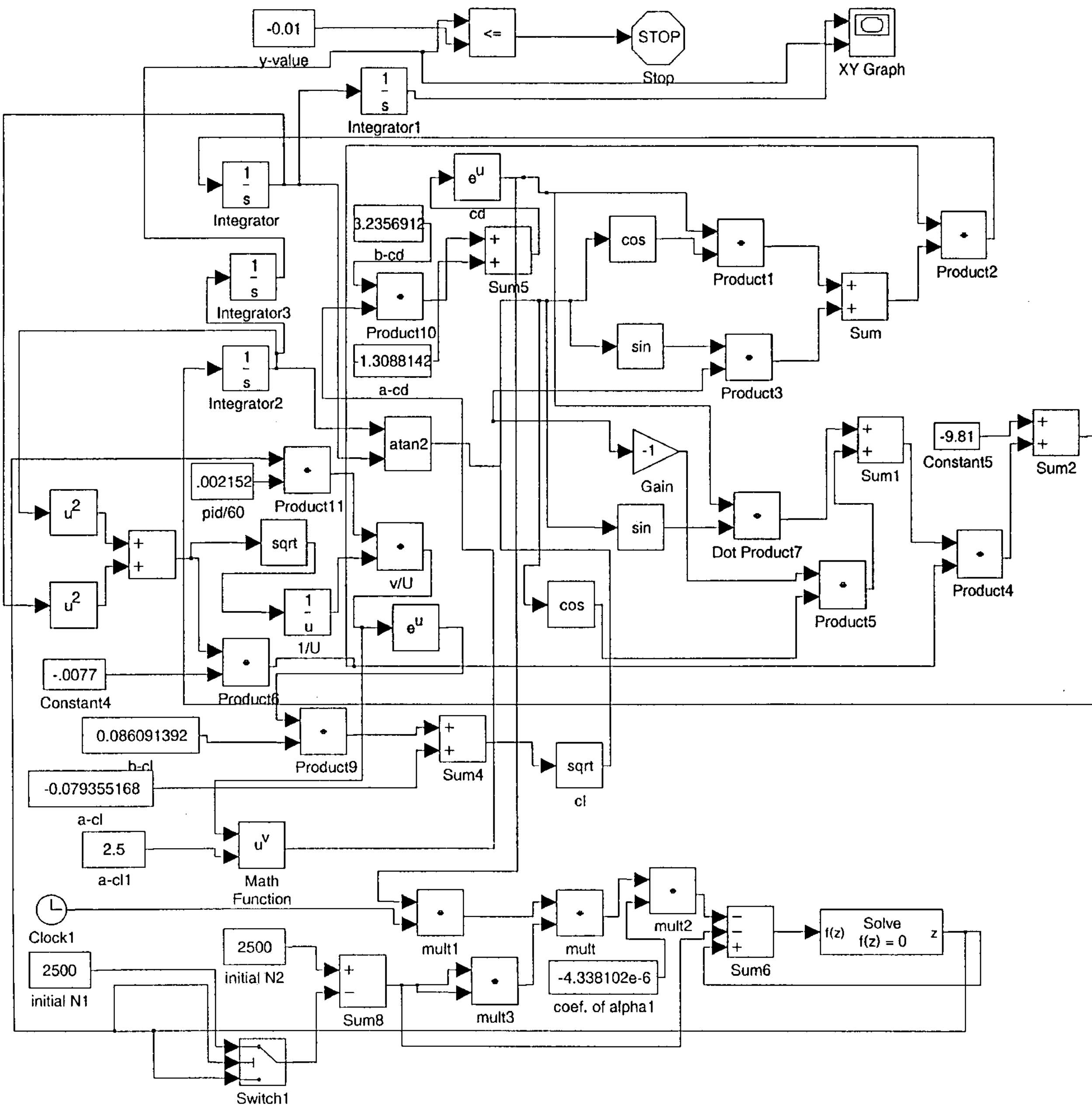

Fig. 2 SIMULINK model.

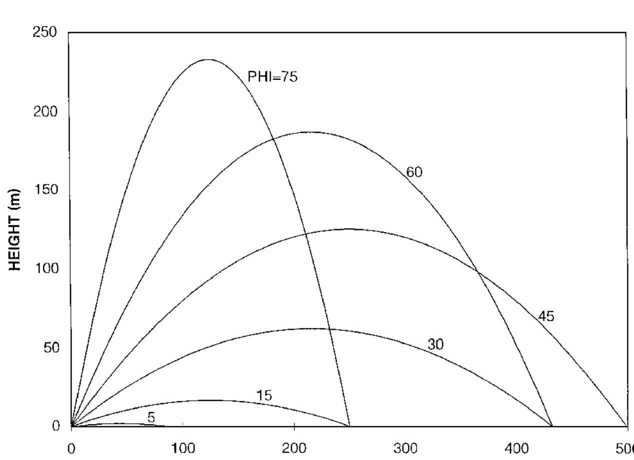

Fig. 3 Trajectory paths in vacuum.

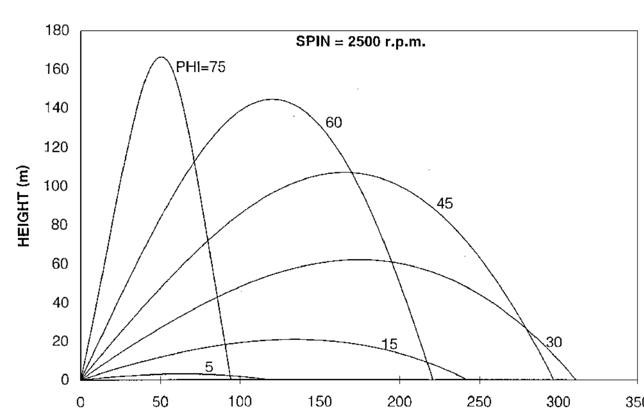

Fig. 4 Trajectory paths with air resistance for initial spin rate of 2500 rpm.

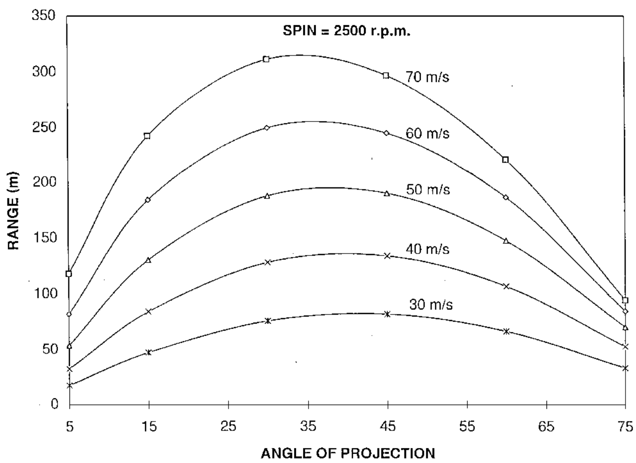

Fig. 5 Effect of maximum range against the angle of projection for different values of initial velocity with initial spin rate of 2500 rpm.

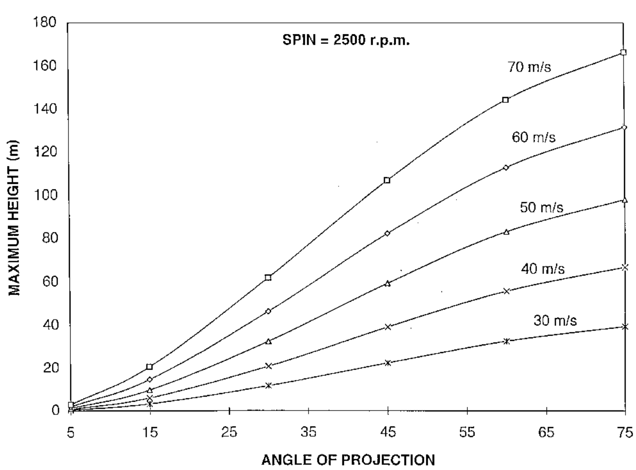

Fig. 6 Effect of maximum height against the angle of projection for different values of initial velocity with initial spin rate of 2500 rpm.

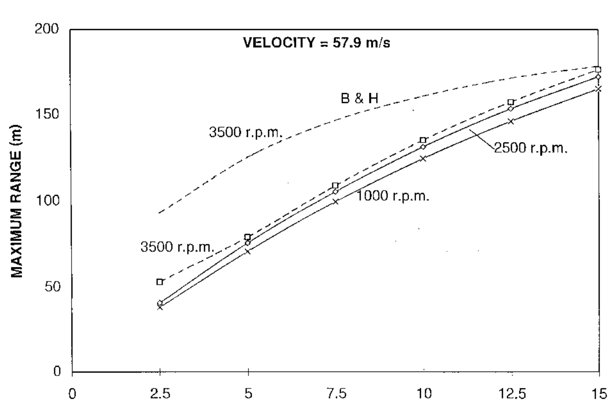

Fig. 7 Effect of maximum range against the angle of projection for different values of spin rates.

ranges for small values of angle of projection, whereas for the angle of projection of 15 deg, the present results agree with those of Bearman and Harvey.[11] Note that they have not considered the spin decay. Maximum range with initial speed are plotted for various spin rates in Fig. 8 while keeping the angle of projection constant at 10 deg. Again the results of Bearman and Harvey for spin rate of 3500 rpm is compared with the present results. It can be seen that present results agree for small velocities and come close for large velocities, whereas in the range of 20–70 m/s, the predicted range by Bearman and Harvey are larger.

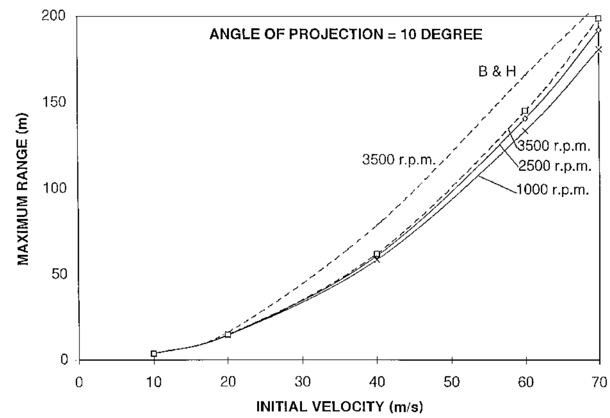

Fig. 8 Effect of maximum range against the initial velocity of projection for different values of spin rates.

Conclusions

Projectile motion in a resistive medium while considering the influence of aerodynamic forces and moments is analyzed. Using aerodynamic data such as the drag and lift coefficients, and also modeling the spin decay, we computed ranges and flight paths for various initial conditions. The effects of initial velocity, angle, and spin are investigated separately. The paper presents a simple mathematical treatment of the problem and its computer implementation using the SIMULINK software. The results clearly demonstrate that the aerodynamic forces and moments significantly affect the range and trajectory of the projectile flight in atmosphere.

References

[1]McShane, E. J., Kelley, J. L., and Reno, F. V., *Exterior Ballistics*, Univ. of Denver Press, Denver, CO, 1953.

[2]Farrar, C. L., and Leeming, D. W., *Military Ballistics*, Brassey, Oxford, 1983.

[3]Hart, D., and Croft, C., *Modelling with Projectiles*, Ellis Horwood, Chichester, England, U.K., 1988.

[4]de Mestre, N. J., "Exterior Ballistics: A Useful Undergraduate Course in Applied Mathematics," *International Journal of Mathematical Education in Science and Technology*, Vol. 8, 1977, pp. 251–257.

[5]Stewart-Townend, M., *Mathematics in Sports*, Ellis Horwood, New York, 1984.

[6]de Mestre, N. J., and Catchpole, T., "Discrete and Continuous Methods in Ballistics," *Int. J. Math. Edn. Sci. Technol.*, Vol. 19, 1988, pp. 155–164.

[7]de Mestre, N. J., *The Mathematics of Projectiles in Sports*, Cambridge Univ. Press, Cambridge, England, U.K., 1990.

[8]Jorgensen, T. P., *The Physics of Golf*, American Inst. of Physics Press, New York, 1994.

[9]Bentley, K., Varty, P., Proudlove, M., and Mehta, R. D., "An Experimental Study of Cricket Ball Swing," Aeronautics TN 82-106, Imperial College, London, 1982.

[10]Davies, J. M., "The Aerodynamics of Golf Balls," *Journal of Applied Physics*, Vol. 20, 1949, pp. 821–828.

[11]Bearman, P. W., and Harvey, J. K., "Golf Ball Aerodynamics," *Aeronautical Quarterly*, Vol. 27, 1976, pp. 112–122.

[12]Mehta, R. D., "Aerodynamics of Sports Balls," *Annual Review of Fluid Mechanics*, Vol. 17, 1985, pp. 151–189.

[13]Murphy, R. V., "Maximum Range Problems in a Resisting Medium," *Math. Gaz.*, Vol. 56, 1972, pp. 10–15.

[14]Parker, G. W., "Projectile Motion with Air Resistance Quadratic in the Speed," *American Journal of Physics*, Vol. 45, 1977, pp. 606–610.

[15]Suzuki, S., and Inooka, H., "A New Golf-Swing Robot Model Utilizing Shaft Elasticity," *Journal of Sound and Vibration*, Vol. 217, 1998, pp. 17–31.

[16]Smits, A. J., and Smith, D. R., "A New Aerodynamic Model of a Golf Ball in Flight," *Science and Golf II Proceedings of the 1994 World Scientific Congress of Golf*, edited by A. J. Cochran, and M. R. Farrally, Chapman and Hall, London, 1994, pp. 340–347.

[17]Adair, R. K., *The Physics of Baseball*, Harper and Row, New York, 1990.

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